MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 96 No. 2

MEETING OF 1935 DECEMBER 13

Mr. J. H. Reynolds, President, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Ibrahim A. N. Abul-Nasr, M.A., 16 Lawrence Street, Glasgow, W. 2 (proposed by T. L. MacDonald) ;

John Grant Bruce, Curator of the Mills Observatory, Dundee, and 66 Nethergate, Dundee (proposed by R. A. Sampson) ; and

Frank Edmund Stafford, O.B.E., F.R.G.S., East India United Service Club, 16 St. James's Square, London, S.W. 1 (proposed by A. R. Hinks).

Seventy-four presents were announced as having been received since the last meeting.

Professor L. M. Milne-Thomson and Mr. W. M. Witchell were appointed Auditors of the Treasurer's accounts for 1935.

NEBULA I.C. II 5146 CYGNUS.

M. de Kerolyr.

The photograph * reproduced in Plate 2 was obtained at the Astrophysical Station of the Paris Observatory at Forcalquier, B.A., 1933 August 19, 20, 21, 22. The total length of exposure was $12^h\ 30^m$, on a "Lumière-Opta" plate. The instrument used was the 80 cm. reflector, focal length 480 cm.

Scale of print : 1 mm. = 15″·6. (Enlarged 2·75 times from the original.)

This nebula is situated in the middle of a region which is apparently void of stars and known as the "dark channel." This "dark channel" is a remarkable absorption cloud easily identified with a pair of binoculars. It shows up easily as a dark area on small scale photographs,† standing out against the lighter background, which is extremely rich in stars of varying brightness.

On the adjoining photograph, taken on a large scale, exactly the reverse is shown. This absorption cloud, this "dark channel," on the contrary, appears grey, and it is only in the surrounding regions and between the numerous bright stars that the background of the sky is shown to be really black.

This absorbing cloud, not merely a region devoid of stars, is thus faintly luminous. Its extreme eastern part is the nebula. It is undoubtedly illuminated by a group of bright stars, of which the most noticeable is of visual magnitude 9·5, the position of which, according to Argelander, is $\alpha = 21^h\ 47^m\ 54^s$; $\delta = +46°\ 34'$ (1855).

NOTE ON THE LATITUDE VARIATION OBSERVATIONS AT GREENWICH IN RELATION TO THE CONSTANT OF ABERRATION AND THE KIMURA TERM.

H. Spencer Jones, F.R.S., Astronomer Royal.

1. The programme of observations with the Cookson floating telescope at the Royal Observatory, Greenwich, was originally planned primarily for the determination of the value of the constant of aberration, the variation of latitude being derived as a by-product of the observations. The observations were therefore planned to be made as near 6^h and 18^h U.T. as possible, in order to obtain the maximum range in the aberration factors. The summer observations are necessarily made near midnight and do not contri-

* This photograph has been reproduced in the R.A.S. series of slides and prints, No. 480 (I 46).

† See *M.N.*, 64, No. 9, where a small-scale photograph by Dr. Max Wolf is reproduced.—SECRETARIES.

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M. de Kerolyr, Nebula I.C. II 5146 Cygnus.

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bute much weight to the aberration ; but they are important for the maintenance of adequate group connections.

The programme comprises 16 groups, each containing 4 or 5 pairs of stars, with the following approximate limits of R.A. :—

Group	Limits of R.A.	Group	Limits of R.A.
	h h		h h
1	0 – 1	9	$14\frac{1}{2}$–16
2	$2\frac{1}{2}$– $3\frac{1}{2}$	10	16 –17
3	$3\frac{1}{2}$– $4\frac{1}{2}$	11	17 –18
4	$7\frac{1}{2}$– $8\frac{1}{2}$	12	18 –19
5	$8\frac{1}{2}$– $9\frac{1}{2}$	13	19 –20
6	11 –12	14	20 –21
7	12 –13	15	21 –22
8	$13\frac{1}{2}$–$14\frac{1}{2}$	16	23 –24

The cyclic arrangement of the groups, forming two interlacing Küdner polygons, which coalesce during the summer months, is as follows :—

Date	Mean Fraction of Year	Evening Groups	Morning Groups
Dec. 25–Jan. 31	0·052	2, 3	6, 7
Feb. 14–Mar. 23	·172	4, 5	8, 9
Mar. 24–Apr. 26	·280	6, 7	9, 10
Apr. 27–May 25	·359	8, 9	10, 11
May 30–June 21	·438	10, 11, 12	
June 22–July 9	·497	11, 12, 13	
July 14–Aug. 7	·566	12, 13	14, 15
Aug. 12–Sept. 19	·671	13, 14	16, 1
Sept. 20–Oct. 25	·756	14, 15	2, 3
Nov. 11–Dec. 22	·918	16, 1	4, 5

It will be observed that there are no direct connections between groups 1 and 2 ; 3 and 4 ; 5 and 6 ; 7 and 8 ; 15 and 16. The group connections are therefore derived by least-squares adjustment of the observations.

For each group and period an equation is obtained of the form

$$z_p + t_q + \alpha_{pq}\kappa = \text{mean residual},$$

where z_p is the mean group correction to the assumed star-places for the pth group, t_q is the latitude variation during the qth period, α_{pq} is the mean aberration factor for this group and period, and $20''{\cdot}47\kappa$ is the correction to the assumed value ($20''{\cdot}47$) of the aberration constant. The mean residual is the mean of the observed *minus* tabular values of the Z.D. north of each star-pair in the group.

Weighting the equations according to the number of observations, normal equations for the z's, the t's and κ are formed. The normal equations for the t's are used to eliminate the t's from the normal equations for the z's. These are then solved and each z derived in terms of κ. Substituting the z values in the normal equations for the t's, each t is derived in terms of κ. Finally substituting for the z's and t's in the normal equation for κ, the value of κ is derived and thence the value of the aberration constant.

2. The constant of aberration is peculiarly difficult to derive free from systematic errors. Its determination is usually based upon the comparison of observations made at opposite seasons of the year, when climatic conditions may be different, and at different times of day. Any error of a seasonal or diurnal nature will therefore affect the derived constant in a systematic manner. The only methods of deriving the constant of aberration that seem to be entirely free from both forms of error are (1) the indirect derivation *via* the solar parallax, and (2) the spectroscopic method, based on the observation of radial velocities of stars.

In the latitude variation programme, errors of a seasonal nature are not much to be feared, but, inasmuch as the constant of aberration is deduced from a comparison between evening and morning observations, any systematic source of error of a diurnal nature will enter directly into the result.

3. The international latitude programme was planned for the determination of the variation of latitude. In this programme there are twelve groups of stars, each extending over approximately two hours of right ascension. The observations are centred around midnight, two groups being observed per night. The same two groups are observed throughout one month ; for the succeeding month the earlier group drops out and the next group is brought in. Thus each group is compared in cyclic rotation with the succeeding group. If I_e, I_m denote the mean latitudes for Group I, derived from evening and morning observations respectively, and similarly for other groups, then the sum

$$(I_e - II_m) + (II_e - III_m) + \ldots + (XII_e - I_m)$$

is free from errors in the adopted latitude variation, as such errors in, for example, the first period affect I_e and II_m similarly ; and, rearranging in the form

$$(I_e - I_m) + (II_e - II_m) + \ldots + (XII_e - XII_m),$$

it is seen to be free from errors in the adopted group corrections, as such errors affect the morning and evening latitudes for the same group equally. This sum is called the "closing error" and, apart from accidental errors, it should be zero.

It is found, however, to be systematically different from zero. In the early years of the latitude variation observations the closing error was interpreted as due to aberration. If the adopted constant of aberration (20″·47) is not correct, errors of opposite sign will be introduced into the evening and morning observations of the same group. This interpretation was abandoned, however, because the mean closing error, derived by combining the results of all stations, led to an impossibly large value of the aberration constant, and because the observations at the individual stations, when considered separately, gave closing errors that differed systematically in amount. Certain stations persistently gave large closing errors.

4. There has been much discussion as to the origin of the closing error. The following possible contributory causes have been considered : (1) an error in the adopted aberration constant ; (2) the neglect in the reductions

of the displacements of the stars arising from annual parallax; (3) the neglect of small nutation terms; (4) forced oscillation of the pole due to the action of the Sun; (5) diurnal change of the plumb line by the Sun; (6) effect of Jupiter and Saturn on aberration.

The total effect from these causes cannot account for more than a small portion of the closing error. It is now generally agreed that the closing error is attributable mainly to local meteorological causes.

5. The influence of meteorological conditions in the Greenwich observations with the Cookson telescope is evidenced by (1) night error, (2) wind effect. There is a tendency for the plates obtained on the same night to give residuals that agree in sign; occasionally, a sequence of plates during one night will all show large residuals in one direction. The effect can be represented as an "error of the night." Discussion of the observations shows that this error amounts to $\pm$ 0″·051 and is the largest contributor to the total probable error of a single plate, $\pm$ 0″·078. The wind effect is shown by a strong correlation between the sign of the plate residuals and the direction of the wind, positive residuals preponderating with north-east winds and negative residuals with south-west winds. The mean difference in the residuals in the two cases is about 0″·10.

6. The arrangement of the Greenwich programme does not permit of closing error being shown so readily as in the programme of the International Latitude Service. Nevertheless, there is a systematic difference between evening and morning observations, which is effectively equivalent to closing error.

From the mean residual (in the sense observed *minus* tabular Z.D. north) for each group and period, the group correction and the latitude variation, derived from the least-squares adjustment, were subtracted. The final residuals so obtained were collected according to group and period of observation. Most groups have two periods of observation, morning and evening; these we denote by *a* and *b*: the groups observed in the summer months have three periods of observation; these we denote by *a*, *b* and *c*.

When the differences between the weighted mean final residuals for the two or three periods of observation of the same group are taken, the results given in the following table are obtained. The mean differences are given for two intervals, 1911–1918 and 1919–1927, and for the whole period over which the results have been discussed, 1911–1927.

Change during Night of Mean Residuals for each Group

Group	Period I 1911–1918	Period II 1919–1927	Combined 1911–1927
	″	″	″
1 *a–b*	−·013	−·053	−·037
2 *a–b*	−·018	−·014	−·016
3 *a–b*	−·005	+·012	+·005
4 *a–b*	+·002	−·037	−·020
5 *a–b*	−·081	+·002	−·030
6 *a–b*	+·008	−·028	−·008
7 *a–b*	−·009	+·012	+·001

Group	Period I 1911–1918	Period II 1919–1927	Combined 1911–1927
	″	″	″
8 *a–b*	−·030	−·009	−·019
9 *a–b*	−·006	−·047	−·025
b–c	−·005	+·024	+·008
a–c	−·010	−·023	−·018
10 *a–b*	+·025	−·062	−·008
b–c	−·034	+·008	−·015
a–c	−·008	−·054	−·023
11 *a–b*	·000	−·023	−·012
b–c	−·017	−·008	−·012
a–c	−·017	−·032	−·024
12 *a–b*	−·017	−·013	−·014
b–c	−·009	−·006	−·008
a–c	−·026	−·019	−·022
13 *a–b*	−·022	−·022	−·022
b–c	−·002	+·005	+·002
a–c	−·024	−·017	−·020
14 *a–b*	−·036	−·026	−·031
b–c	+·003	+·013	+·009
a–c	−·034	−·013	−·022
15 *a–b*	−·002	−·015	−·008
16 *a–b*	−·070	+·017	−·017

Using the extreme differences for Groups 9 to 14, the sum of the differences (morning *minus* evening) for the sixteen groups becomes

$$-0''\cdot337 \text{ for } 1911–1918$$
$$-0''\cdot271 \text{ for } 1919–1927$$
$$-0''\cdot278 \text{ for } 1911–1927$$

The sum is analogous to the closing error that enters into the reductions of the international latitude variation results, and corresponds in sign: for we have dealt with residuals in north zenith distance $(\delta - \phi)$; interpreted as a systematic difference between latitude deduced from evening and morning observations, our result implies that the latitude is apparently systematically greater in the morning than in the evening. This agrees with the systematic difference inferred from the closing error in the international results.

7. The systematic difference between morning and evening observations has been interpreted, in the discussion of the Greenwich observations, as due to the employment of an erroneous value of the constant of aberration in the reductions. The above closing errors lead at once to the corrections to the aberration constant. The mean differences in the residuals (morning *minus* evening) per group are $-''\cdot021$ and $-''\cdot017$ respectively for the two periods. The mean differences in aberration factor (morning *minus* evening) for any group do not greatly differ from 16. It follows that the corrections to the aberration factor for the two periods are

$$\frac{20''\cdot47}{16} \times (+\cdot021 \text{ or } +\cdot017).$$

The corrections for the two periods are thus +″·027 and ″·022. Those derived from the more elaborate least-squares solution were ″·028 and ″·023,* in close agreement with the preceding corrections.

Since the closing error in the international latitude programme cannot be explained as arising from an erroneous value of the adopted constant of aberration, and since the systematic difference between the morning and evening observations at Greenwich is essentially the same in nature and in the same sense as the closing error, the correction to the constant of aberration derived from the Greenwich latitude variation observations must be regarded as spurious.

8. If there is a variable term of the form $f(\odot, \alpha)$ in the observed latitude, where $\odot$ denotes the longitude of the Sun and α denotes right ascension, then the change in α in the course of a night will be interpreted as an apparent motion of the pole during the night, of a diurnal nature, whilst the change in $\odot$ in the course of the year will be interpreted as an apparent annual motion of the pole. It is in this way that closing error, or a systematic difference between latitudes derived from evening and morning observations, is related to the z term in the latitude variation.

9. H. Kimura has shown, in several papers,† that the evening and morning differences in latitude and the annual term in the latitude variation can be reasonably well represented by a combination of terms of the forms $\cos(\odot - \alpha)$, $\sin(\odot - \alpha)$, $\cos(2\odot - \alpha)$ and $\sin(2\odot - \alpha)$. In his recent discussion of the international latitude observations ‡ he has considered, in addition, terms involving the sine and cosine of α, $\odot$, $2\odot$ and $(2\odot - 2\alpha)$. As not all of these terms could be separated from each other, it was necessary to make some rather arbitrary assumptions.

10. It may be noted that some of the neglected theoretical terms, which may contribute something to the annual or z term in the latitude variation, involve some of the arguments considered by Kimura. An error in the adopted aberration constant brings in terms involving $\cos\odot$, $\sin(\odot - \alpha)$; the neglect of the mean parallactic displacements of the stars involves $\sin\odot$ and $\cos(\odot - \alpha)$; the fixed oscillation of the pole due to the action of the Sun involves $\sin\alpha$ and $\sin(2\odot - \alpha)$; the diurnal change in the vertical due to the Sun involves $\cos 2(\odot - \alpha)$, etc. Such terms will combine with terms of similar general form arising from diurnal or annual variations of meteorological origin.

11. If we suppose that there is a term of the form $f(\odot, \alpha)$ in the observed

* The corrections to the aberration constant, deduced from the Greenwich observations with the Cookson floating telescope, have been given with the wrong sign for both periods, 1911–1918 and 1919–1927, in the original publications. It is easily seen that, since the aberration factors have been taken in the sense of a correction to apparent declination to give true declination, and since the residuals used are in the sense observed *minus* tabular, the sign of the κ term should be changed in the equations of condition used.

† E.g. *Japanese Journal of Astronomy and Geophysics*, 1, No. 1, 1922; *Travaux de la Section de Géodésie de l'Union Géodésique et Géophysique Internationale*, tome 5 (Report presented by the Japanese National Committee), 1927.

‡ *Results of the International Latitude Service*, 7 (1922·7 to 1931·0), 1935.

latitude then, in the reduction of the observations at Greenwich with the Cookson floating telescope, the correct equation of condition to use for observations of the pth group during the qth period is

$$z_p + t_q + f(\odot, \alpha) = \text{mean residual.}$$

Normal equations can be formed in the usual way for the z's and the t's and solved to derive z, the group correction, for each group and t, the latitude variation, for each period. z_p will include a term of the form $\phi_p(\odot, \alpha)$ and t_q will include a term of the form $\psi_q(\odot, \alpha)$.

The residual, for the pth group and qth period, will therefore involve

$$f(\odot, \alpha) + \phi_p(\odot, \alpha) + \psi_q(\odot, \alpha).$$

The question arises whether from an analysis of the residuals it is possible to determine any terms of the form $f(\odot, \alpha)$, which are present in the observed latitude.

12. Terms have been considered involving the sine and cosine of the following arguments : $\odot$, α, $(\odot - \alpha)$, $(2\odot - \alpha)$ and $(2\odot - 2\alpha)$. The approximate effect of such terms was investigated, using as a basis the detailed results given by the Cookson floating telescope for the years 1911–1918. In order to reduce the amount of arithmetical computation, it was assumed (1) that the mean date of the observations of any one group during any given period was the same in each year ; (2) that the total observations during any given period were divided amongst the three or four groups, observed during that period, in the same proportion each year.

The observations during one year are divided into 10 periods, as described in § 1. We denote these in order by $t_1, t_2, \ldots t_{10}$. The mean effects on the t's and the z's of terms of different type are summarised in the table opposite. The results are given in the following sense. If in the equations of condition there are terms

$$c \cos (m\odot - n\alpha) + s \sin (m\odot - n\alpha),$$

then each t and z obtained by solving the normal equations for the t's and z's will contain terms of the form

$$\lambda c + \mu s.$$

The values of λ, μ are tabulated for the arguments $\odot$, α, $(\odot - \alpha)$, $(2\odot - \alpha)$, $(2\odot - 2\alpha)$ in the following table.

It will be noted that in each case the result of the process of least-squares adjustment is to modify the effect of terms such as we have supposed to be present in the observed latitude, so that the derived latitudes are affected by an annual variation, having a single maximum and a single minimum.

13. The next step was to insert the derived values of the t's and z's, for any of the above arguments, into the equations of condition, to obtain the mean residuals for each period and group, and to analyse these by least-squares solution in order to determine the coefficients. In general, however, no definite results could be obtained.

The terms with argument $\odot$ do not enter into the final residuals and

Values of λ, μ for Various Arguments

Argument	$\odot$		α		$\odot - \alpha$		$2\odot - \alpha$		$2\odot - 2\alpha$	
	λ	μ	λ	μ	λ	μ	λ	μ	λ	μ
t_1	−·52	+·86	+ ·04	+ ·24	+ ·37	+·25	+ ·50	−1·04	+ ·79	−·11
t_2	−·96	+·27	− ·06	+ ·24	+ ·60	+·37	+1·32	− ·51	+ ·21	−·26
t_3	−·94	−·35	− ·01	+ ·18	+ ·84	+·52	+1·63	+ ·38	− ·50	−·33
t_4	−·67	−·74	− ·01	+ ·18	+ ·99	+·51	+1·29	+1·24	− ·93	−·42
t_5	−·23	−·97	+ ·02	+ ·18	+1·11	+·35	+ ·45	+1·80	−1·39	−·38
t_6	+·16	−·99	+ ·02	+ ·19	+1·14	−·07	− ·41	+1·82	−1·45	−·06
t_7	+·56	−·83	+ ·06	+ ·22	+1·09	−·43	−1·20	+1·40	−1·23	+·27
t_8	+·95	−·32	+ ·07	+ ·22	+ ·90	−·43	−1·67	+ ·25	− ·51	+·15
t_9	+·99	+·26	+ ·07	+ ·26	+ ·65	−·37	−1·17	− ·46	+ ·15	+·30
t_{10}	+·39	+·92	− ·01	+ ·25	+ ·31	−·15	− ·37	−1·03	+1·02	+·16
z_1	nil	nil	−1·05	− ·35	+ ·08	+·59	+1·01	+ ·48	− ·29	−·30
z_2			− ·79	−1·00	+ ·03	+·18	+ ·37	+1·09	− ·21	+·19
z_3			− ·55	−1·16	+ ·05	+·07	+ ·48	+1·10	− ·09	−·06
z_4			+ ·57	−1·14	+ ·21	−·14	− ·51	+ ·85	− ·51	−·01
z_5			+ ·85	− ·99	+ ·16	−·24	− ·44	+ ·88	− ·57	−·14
z_6			+1·01	− ·28	+ ·09	−·36	− ·97	+ ·31	− ·19	+·02
z_7			+ ·99	·00	+ ·11	−·66	− ·85	+ ·13	− ·33	+·32
z_8			+ ·90	+ ·30	+ ·02	−·57	− ·83	− ·34	+ ·02	+·41
z_9			+ ·70	+ ·57	− ·01	−·79	− ·75	− ·45	+ ·06	+·75
z_{10}			+ ·41	+ ·74	− ·11	−·64	− ·52	− ·78	+ ·40	+·65
z_{11}			+ ·15	+ ·81	− ·15	−·40	− ·19	− ·92	+ ·54	+·45
z_{12}			− ·16	+ ·81	− ·18	+·06	+ ·22	− ·92	+ ·58	+·02
z_{13}			− ·48	+ ·72	− ·17	+·58	+ ·59	− ·76	+ ·50	−·47
z_{14}			− ·67	+ ·56	− ·12	+·92	+ ·76	− ·52	+ ·26	−·85
z_{15}			− ·85	+ ·37	− ·04	+·80	+ ·70	− ·43	+ ·11	−·78
z_{16}			−1·03	+ ·04	+ ·03	+·60	+ ·95	+ ·29	− ·31	−·18

are therefore indeterminate. The terms with argument α give residuals so small that the probable errors of the coefficients derived from them are many times greater than the coefficients themselves, which are therefore without significance.

The terms with arguments $(\odot - \alpha)$, $(2\odot - \alpha)$, $(2\odot - 2\alpha)$ must be considered together. The residuals are so small that the coefficients cannot be determined with any accuracy, except for the terms depending upon sin $(\odot - \alpha)$ and sin $(2\odot - 2\alpha)$. For either of these terms the residuals are large enough for an accurate determination of the coefficients to be possible, but the correlation between the residuals for the two terms is so strong that the determination of both becomes impossible. The correlation is produced by cos $(\odot - \alpha)$ having the same sign for both morning and evening groups and a limited range in numerical value.

Thus if terms of the form sin $(\odot - \alpha)$ and sin $(2\odot - 2\alpha)$ are present in the observed latitudes, they cannot be separated in the programme of observations with the Cookson telescope. The observations can be

represented equally well on the hypothesis that either term is present; we shall therefore make the assumption that a term in sin $(\odot - \alpha)$ is present.

The terms depending upon cos $(\odot - \alpha)$, cos $(2\odot - \alpha)$, sin $(2\odot - \alpha)$ and cos $(2\odot - 2\alpha)$ cannot be determined with much accuracy, as their coefficients enter with small weight into the final residuals. There is, moreover, considerable correlation between the terms depending on cos $(\odot - \alpha)$, sin $(2\odot - \alpha)$ and cos $(2\odot - 2\alpha)$, whilst the cos $(2\odot - \alpha)$ term is correlated with the terms depending upon sin $(\odot - \alpha)$ and sin $(2\odot - 2\alpha)$. The probable error of the coefficient of each of these terms is larger than the coefficient itself.

14. The conclusion from this investigation is that the programme of observations does not permit of the detection of any of the terms that Kimura supposes to be present in the observed latitudes, with the exception of a term of the form sin $(\odot - \alpha)$. This term has opposite signs for the evening and morning observations. It is the presence of a term in the aberration factor depending upon sin $(\odot - \alpha)$ that permits of the observations being satisfactorily represented by a correction to the aberration factor.

The coefficient of the sin $(\odot - \alpha)$ term was obtained in two ways: (*a*) from an analysis of the residuals for each group and period; (*b*) by taking the differences of morning and evening residuals for each group and analysing these. Any errors in the adopted group corrections enter into (*a*), but are eliminated from (*b*). The results derived from the analysis of the observations for the period 1911–1927 give for the coefficient of the sin $(\odot - \alpha)$ term (on the assumption that terms of the form cos $(\odot - \alpha)$, cos $(2\odot - \alpha)$, etc., are not present) the values

$+''\cdot 0164$ by method (*a*),
$+''\cdot 0159$,, ,, (*b*).

These values are changed only by a few units in the fourth place of decimals by the inclusion of the other terms in the solutions.

The term $+''\cdot 016 \sin(\odot - \alpha)$ in the equations of condition (whose right-hand sides are observed *minus* tabular Z.D. north) is equivalent to a term $-''\cdot 016 \sin(\odot - \alpha)$ in the latitude. But the term does not appear to the full amount in the deduced latitudes, because of the least-squares adjustment. The effect on the deduced latitudes is $-''\cdot 016\,\mu$, where μ is to be taken from the table above, under argument $(\odot - \alpha)$.

Interpolating to tenths of a year, the corresponding term in the latitude variation is:

(Unit $0''\cdot 001$)

Fraction of year .	·1	·2	·3	·4	·5	·6	·7	·8	·9
$-''\cdot 016\,\mu$ . . −2	−5	−7	−8	−7	0	+7	+7	+6	+2

15. The conclusion of this investigation is that the observed latitude at Greenwich contains a term that can be represented by $-''\cdot 016 \sin(\odot - \alpha)$. There may be terms depending upon the sine or cosine of $\odot$, α, $2\odot - \alpha$, $2\odot - 2\alpha$ and upon cos $(\odot - \alpha)$. With the exception of the term sin $(2\odot - 2\alpha)$, these are almost entirely eliminated in the least-squares adjustment and, unless their coefficients are large, they will not appreciably affect the deduced latitudes.

The term in sin $(\odot - \alpha)$ changes sign at midnight and causes a variation in the observed latitude during the course of the night, which has been interpreted as an aberration effect and as giving a correction to the adopted value (20″·47) of the constant of aberration. For the reasons given above, this interpretation must be discarded.

In the course of the year, the term in sin $(\odot - \alpha)$ produces an annual term in the deduced latitude variations. The amount of this variation is given at the end of § 14. The total amplitude is 0″·015, the minimum value being reached at the fraction 0·3 of the year and the maximum value at the fraction 0·65.

The Kimura term at Greenwich (obtained by subtracting the x component of the international latitude results from the Greenwich observed latitude variation) has a minimum at about 0·4 and a maximum at about 0·7. The minimum and maximum of the Greenwich Kimura term therefore coincide very closely with those of the latitude variation caused by the sin $(\odot - \alpha)$ term. It must be concluded that this term contributes to the z or Kimura term in the Greenwich results.

16. It has been shown by S. Kawasaki * that the systematic effect of wind on the observed latitudes also contributes to the z term. His detailed discussion of the observations for the period 1919–1927 shows that the double maximum and double minimum in the z term for that period † can thus be accounted for. The principal minimum and maximum occur respectively at the fractions of the year 0·3 and 0·7, and the secondaries at 0·8 and 0·1 respectively. The times of both principal minimum and maximum of the term due to the wind effect, of the sin $(\odot - \alpha)$ term and of the Kimura term are therefore in close agreement. The amplitudes (measured by the range from principal minimum to principal maximum) are :

	Range
sin $(\odot - \alpha)$ term .	″·015
Wind effect . .	·024
z term ‡ . .	·043

The sin $(\odot - \alpha)$ term and the wind effect jointly can therefore satisfactorily account for the full amplitude of the Greenwich z term, as well as for the phase variations.

The detailed comparison is as follows :—

(Unit 0″·001)

Fraction of year	0	·1	·2	·3	·4	·5	·6	·7	·8	·9
Combined terms	+11	+ 8	−15	−18	−16	−9	+20	+21	−3	− 3
Greenwich z term	+ 6	+13	+ 4	−12	−26	−9	+15	+17	0	−12

The agreement is as close as could reasonably be expected, for there are probably other small factors contributing to the z term.

* *M.N.*, 95, 33, 1934.

† For the period 1911–1918 the secondary maximum and minimum did not appear in the Greenwich z term.

‡ As given by Kimura, *Trans. I.A.U.*, 3, p. 122.

MEAN STELLAR PARALLAXES DERIVED FROM PROPER MOTIONS.

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(Communicated by the Director, Cambridge Observatory)

1. The observational measures used in this investigation consist of the total proper motions (photographic) of the B.D. stars occurring in the eight groups of regions which I analysed some years ago on the basis of the two-streams theory.* The components of the proper motion of each star in the various photographic regions are given in *Cambridge Astronomical Observations*, **26**, 1928, and from these the values of the total proper motion μ have been obtained.† We are not concerned here with the position angles of the proper motions but only with the *numerical* values of μ. The B.D. stars have been arranged in three magnitude groups : (*a*) $7^{m}\cdot0$–$8^{m}\cdot9$, (*b*) $9^{m}\cdot0$–$9^{m}\cdot4$ and (*c*) $9^{m}\cdot5$; the mean B.D. magnitude for a particular region and magnitude division is subsequently reduced to the Harvard scale according to a table of corrections by Kapteyn and van Rhijn.‡

In deriving photographic proper motions it may be recalled that the reduction of the plate-measures first of all gives the relative proper motions of the stars, and these are almost invariably converted into absolute proper motions by means of the parallactic motions which involve a knowledge of the mean parallaxes of the stars selected for this purpose. In reducing the Cambridge proper motions I used, in each field investigated, the mean parallactic motions of the B.D. stars (arranged in several magnitude groups) as calculated from the secular parallaxes given in *Groningen Publications*, **29**, Table XXVI, and I compared these with the mean *algebraic* relative proper motions as obtained from the plate-measures. Here we consider the *numerical* values of the total proper motions of the same stars and from them derive the mean parallaxes. It may appear that this procedure is open to criticism, but in using the assumed values of the secular parallaxes to derive the correction for converting relative proper motions into absolute proper motions we may regard such assumed values as being of the nature of a first approximation. An uncertainty of 10 per cent. in the secular parallaxes may result in an uncertainty of $0''\cdot001$ in the value of the annual proper motion correction in each co-ordinate, but it will have a negligible effect on the distribution of the total proper motions so far as the magnitude of these is concerned.

2. The method adopted in this investigation for deriving mean parallaxes from total proper motions is one I suggested in a recent paper.§ It is based on the two-streams theory and, accordingly, a knowledge of the stream-

* *M.N.*, **87**, 122, 1926.

† I am indebted to Mr. R. A. Lyttleton, B.A., of Clare College, for the greater part of these calculations.

‡ *Groningen Publications*, **30**, 12, 1920.

§ *M.N.*, **95**, 128, 1934.

constants is essential. The mean linear tangential velocity, ρ, of the stars in a given small area of the sky is given by

$$\rho = \bar{T}\{\alpha\psi(b_1) + (1-\alpha)\psi(b_2)\}, \tag{1}$$

where α and $(1-\alpha)$ are the proportions of the stars in the two drifts, $\bar{T}$ is the mean random linear tangential velocity for a drift, b_1 and b_2 are respectively $\frac{1}{2}(hv_1)^2$ and $\frac{1}{2}(hv_2)^2$, where hv_1 and hv_2 are the tangential components of the drift velocities, relative to the sun, in terms of a theoretical unit $1/h$, and ψ is a function whose values are tabulated in the paper just referred to. The complete analysis of the eight composite regions has been given * in an earlier paper, and the values of hv_1, hv_2 and α to be used in formula (1) are taken from Table VI (columns 3 and 6) and § 9 of that paper.

Also, the solar motion was found to be 0·881 in terms of the unit $1/h$, and identifying this value with the generally adopted value of 19·5 km./sec., we obtain $1/h$ expressed in the usual units. Again,† $\bar{T} = \sqrt{\pi}/2h$, so that

$$\bar{T} = \frac{\sqrt{\pi}}{2} \cdot \frac{19{\cdot}5}{0{\cdot}881} = 19{\cdot}6 \text{ km./sec.}$$

If we assume that all the stars of a magnitude group have the same parallax p, we have—denoting the mean observed total proper motion by $\bar{\mu}$—

$$\rho = 4{\cdot}74\frac{\bar{\mu}}{p}.$$

Hence, using the above value of $\bar{T}$,

$$p = 0{\cdot}242\frac{\bar{\mu}}{D}, \tag{2}$$

where

$$D = \alpha\psi(b_1) + (1-\alpha)\psi(b_2). \tag{3}$$

Abandoning the assumption that all the stars of the particular magnitude group concerned are at the same distance, we regard the value of p, derived from (2), as the statistical mean parallax. The observations furnish the appropriate values of $\bar{\mu}$, and the corresponding values of p are then easily calculated.

3. Before proceeding to make the simple calculations for p by means of (2), it is necessary to correct the proper motions for the effect of accidental errors. The measures of the relative proper motions provide a means of estimating the probable error r in each co-ordinate, and to obtain the effect of the errors on the total proper motions it is essential to know the distribution of the observed total proper motions, which we now denote by μ_0. It is convenient, for the correction of the statistics, to express the proper motions μ_0 as centennial proper motions. Tables I, II and III give the observed distribution for each of the magnitude divisions and for each of the eight groups of regions I to VIII, the entries being the number of total proper motions μ_0 between 0″·0 and 0″·5, 0″·5 and 1″·0, etc. A star with $\mu_0 = 2''{\cdot}5$, for example, contributes $\frac{1}{2}$ to the interval 2″·0–2″·5 and $\frac{1}{2}$ to the

* *M.N.*, **97**, 136, 1926.
† *M.N.*, **95**, 120 (equation 24), 1934.

interval 2″·5–3″·0. In the penultimate line of each of the tables the total number of stars in each group of regions is given, and in the last line the sums of the observed total proper motions are given. The co-ordinates of the centres of the eight regions, together with the corresponding galactic latitudes, are to be found in *M.N.*, **87**, 124, 1926.

TABLE I

Distribution of Total Proper Motions ($7^m·0$–$8^m·9$)

μ_0	I	II	III	IV	V	VI	VII	VIII
″								
0·0	0	3	1	1	0	2	5	½
0·5	3½	16	3	6½	2	7	3½	1
1·0	6½	13	6	8½	1	7	7	½
1·5	5½	9	5½	7½	2	5	9	2
2·0	6	7½	4	6½	1	7	4½	4½
2·5	1	11	2½	2	1	4½	4½	4½
3·0	2	5	3	0	1	1	2½	1½
3·5	2½	4	2	1	½	2½	1	½
4·0	3	1½	3	1	1½	1	2	0
4·5	1	0	1	0	0	0	0	0
5·0	0	2½	0	0	0	½	0	0
5·5	0	2½	3	½	2	½	1	0
6·0	1	8	4	3½	5	2	5	2
> 6·0								
Total Number of Stars	32	83	38	38	17	40	45	17
$\Sigma\mu_0$	83″·8	266″·4	138″·5	92″·8	80″·4	126″·0	123″·9	57″·2

TABLE II

Distribution of Total Proper Motions ($9^m·0$–$9^m·4$)

μ_0	I	II	III	IV	V	VI	VII	VIII
″								
0·0	1½	5	2½	4	0	11	9½	½
0·5	8½	12	8½	12½	½	16½	11	7
1·0	6½	26	12½	21	4½	18	9	8
1·5	11	16	9½	13½	1	16½	7	10½
2·0	9½	18	7½	9	3	10	2½	8

μ_0	I	II	III	IV	V	VI	VII	VIII
″								
2·5								
	6	8½	6	4½	1½	5	3½	3
3·0								
	3½	7½	2½	6	3½	2½	6	1
3·5								
	1½	2	6	4½	1	2	3½	2
4·0								
	2½	6	1½	1	1	1½	1	1
4·5								
	½	0	1	1	3	0	1	0
5·0								
	1½	2	3½	½	3	0	1	0
5·5								
	3	4	2	1½	0	0	½	½
6·0								
> 6·0	3½	8	5	3	4	3	½	4½
Total Number of Stars	59	115	68	82	26	86	56	46
$\Sigma\mu_0$	159″·8	297″·3	188″·7	179″·9	107″·9	163″·8	102″·2	113″·1

TABLE III

Distribution of Total Proper Motions ($9^m·5$)

μ_0	I	II	III	IV	V	VI	VII	VIII
″								
0·0								
	1	4	4	6	0	6	½	1½
0·5								
	5	13	8	14	0	9½	8½	5
1·0								
	6	12	5	18	1½	14	12	10
1·5								
	9	12½	5	9½	3½	10	4	5
2·0								
	8	9½	2½	11	3	10½	4	6½
2·5								
	3	10	6	7	1½	3½	2	5
3·0								
	5	1	2½	5½	3½	2½	1½	2½
3·5								
	1	1	0	5½	1	0	½	½
4·0								
	2	1	0	1½	½	0	½	0
4·5								
	0	0	0	2	1½	0	1½	0
5·0								
	0	1	1	0	1	0	1	0
5·5								
	0	0	½	0	1	0	1	0
6·0								
> 6·0	1	3	1½	4	5	4	1	3
Total Number of Stars	41	68	36	84	23	60	38	39
$\Sigma\mu_0$	89″·7	151″·7	70″·7	176″·8	88″·2	120″·7	79″·3	83″·7

4. Kapteyn and van Rhijn have given two methods * for correcting the total proper motions.

Let $f(\mu)d\mu$ denote the proportion of stars with true total proper motions between μ and $\mu + d\mu$, and let $\phi(\mu_0)d\mu_0$ denote the proportion of observed total proper motions between μ_0 and $\mu_0 + d\mu_0$. If h is the modulus for the Gaussian law of errors and r is the probable error, h is calculated from

$$h = \frac{0{\cdot}477}{r}. \tag{4}$$

It is shown that, for equal values of r in each co-ordinate,

$$\phi(\mu_0) = \frac{2h^2}{\pi}\mu_0 e^{-h^2\mu_0^2}\int_0^\infty f(\mu)e^{-h^2\mu^2}d\mu\int_0^\pi e^{2h^2\mu\mu_0\cos\theta}d\theta,$$

which may be written in terms of the Bessel function of zero order,

$$\phi(\mu_0) = 2h^2\mu_0 e^{-h^2\mu_0^2}\int_0^\infty f(\mu)e^{-h^2\mu^2}J_0(i\,.\,2h^2\mu\mu_0)d\mu. \tag{5}$$

If

$$f(\mu) = 2\beta^2\mu e^{-\beta^2\mu^2}, \tag{6}$$

it is found that

$$\phi(\mu_0) = 2k^2\mu_0 e^{-k^2\mu_0^2}, \tag{7}$$

where

$$k = \frac{h\beta}{(h^2+\beta^2)^{1/2}}.$$

It follows that if the observed distribution is given by (7), the true distribution is given by (6), where

$$\beta = \frac{hk}{(h^2-k^2)^{1/2}}. \tag{8}$$

The mean value $\bar{\mu}$ of the proper motions in the true distribution and the mean value $\bar{\mu}_0$ of the proper motions in the observed distribution are respectively

$$\bar{\mu} = \frac{\sqrt{\pi}}{2\beta}, \qquad \bar{\mu}_0 = \frac{\sqrt{\pi}}{2k}. \tag{9}$$

It is to be noticed that here $\bar{\mu}_0$ denotes the mean value of $\bar{\mu}_0$ in the distribution defined by (7).

If k can be determined from the distribution actually observed, β is easily found from (8).

In practice it is more convenient to determine the value of k from the curve

$$y = \Phi(\mu_0),$$

---

* *Groningen Publications*, **30**, 43, 61, 1920.

where $\Phi(\mu_0)$ denotes the proportion of observed total proper motions greater than a given value μ_0. From (7),

$$\Phi(\mu_0) = \int_{\mu_0}^{\infty} \phi(\mu_0) d\mu_0 = e^{-k^2\mu_0^2}. \tag{10}$$

In nearly every instance, so far as this paper is concerned, the distribution of the observed total proper motions leads approximately to a function of type (10), or to a combination of two such functions with moduli k_1 and k_2 and factors A_1 and A_2.

The method first involves the calculation of the correction c given by

$$c = \frac{\sqrt{\pi}}{2}\left(\frac{1}{k} - \frac{1}{\beta}\right); \tag{11}$$

or in the more general case,

$$c = \frac{\sqrt{\pi}}{2}\left(\sum A_n\left(\frac{1}{k_n} - \frac{1}{\beta_n}\right)\right), \tag{12}$$

with the condition $\Sigma A_n = 1$.

5. In the second method, the mean value $\bar{\mu}_1$ is found of the true total proper motions of the stars whose observed total proper motion is a given value μ_0. Taking the general case where the observed distribution can be represented by several functions of the form (7) and dropping the suffixes from A_n, β_n, we have

$$\bar{\mu}_1 = \frac{\sum A\beta^2 \int_0^{\infty} \mu^2 e^{-\mu^2(h^2+\beta^2)} J_0(i \,.\, 2h^2\mu\mu_0) d\mu}{\sum A\beta^2 \int_0^{\infty} \mu e^{-\mu^2(h^2+\beta^2)} J_0(i \,.\, 2h^2\mu\mu_0) d\mu}. \tag{13}$$

For a single function of type (7) Kapteyn and van Rhijn obtain

$$\bar{\mu}_1 = \frac{G(P)}{(h^2+\beta^2)^{1/2}},$$

where

$$P = \frac{h^4\mu_0^2}{h^2+\beta^2}, \tag{14}$$

and $G(P)$ is expressed as an infinite series which is rapidly convergent for $P < 1$; for $P > 1$, the calculation of $G(P)$ is effected by numerical integration. The tabulation of the function is thus very laborious.

It will now be shown that $G(P)$ can be expressed simply in terms of known functions and is actually $\frac{\sqrt{\pi}}{2}\psi\left(\frac{P}{2}\right)$, where ψ is the function in (1).

We require the following well-known formulæ:—

$$\int_0^{\infty} x^{2n+1} e^{-H^2x^2} dx = \frac{1}{2} \cdot \frac{n!}{H^{2n+2}}, \tag{15}$$

$$\int_0^\infty x^{2n}e^{-H^2x^2}dx = \frac{\sqrt{\pi}}{2}\cdot\frac{1\cdot 3\cdot 5\ldots 2n-1}{2^n}\cdot\frac{1}{H^{2n+1}}, \tag{16}$$

$$\mathcal{J}_0(\xi) = \sum_0^\infty (-1)^n\frac{\xi^{2n}}{2^{2n}(n!)^2}. \tag{17}$$

Consider an integral K_1 of the denominator of (13), so that

$$K_1 = \int_0^\infty \mu e^{-H^2\mu^2}\mathcal{J}_0(\mu y)d\mu, \tag{18}$$

in which

$$H^2 = h^2+\beta^2 \quad \text{and} \quad y = 2h^2\mu_0 i. \tag{19}$$

Then by (17)

$$K_1 = \int_0^\infty \sum_0^\infty (-1)^n\frac{y^{2n}}{2^{2n}(n!)^2}\mu^{2n+1}e^{-H^2\mu^2}d\mu,$$

from which, using (15),

$$K_1 = \frac{1}{2H^2}\sum_0^\infty (-1)^n\left(\frac{y^2}{4H^2}\right)^n\cdot\frac{1}{n!}$$

$$= \frac{1}{2H^2}e^{-y^2/4H^2};$$

or, by means of (14),

$$K_1 = \frac{e^P}{2H^2}. \tag{20}$$

Consider now an integral K_2 of the numerator of (13), so that

$$K_2 = \int_0^\infty \mu^2 e^{-H^2\mu^2}\mathcal{J}_0(\mu y)d\mu$$

$$= \int_0^\infty \sum_0^\infty (-1)^n\frac{y^{2n}}{2^{2n}(n!)^2}\mu^{2n+2}e^{-H^2\mu^2}d\mu,$$

and, using (16),

$$K_2 = \frac{\sqrt{\pi}}{2}\cdot\sum_0^\infty (-1)^n\cdot\frac{y^{2n}}{2^{2n}(n!)^2}\cdot\frac{1\cdot 3\cdot 5\ldots 2n+1}{2^{n+1}}\cdot\frac{1}{H^{2n+3}}$$

$$= \frac{\sqrt{\pi}}{4H^3}\sum_0^\infty P^n\frac{2n+1}{n!}\cdot\frac{1\cdot 3\cdot 5\ldots 2n-1}{2\cdot 4\cdot 6\ldots 2n}.$$

But

$$\frac{1\cdot 3\cdot 5\ldots 2n-1}{2\cdot 4\cdot 6\ldots 2n} = \frac{1}{\pi}\int_0^\pi \cos^{2n}\theta d\theta.$$

Hence

$$K_2 = \frac{1}{4\sqrt{\pi}H^3}\int_0^\pi\left\{\sum_0^\infty (1+2n)\cdot\frac{(P\cos^2\theta)^n}{n!}\right\}d\theta$$

$$= \frac{1}{4\sqrt{\pi}H^3}\int_0^\pi (1+2P\cos^2\theta)e^{P\cos^2\theta}d\theta;$$

or, writing $P=2b$,

$$K_2=\frac{e^{2b}}{4\sqrt{\pi}H^3}\int_0^{\pi}(1+4b\cos^2\theta)e^{-2b\sin^2\theta}d\theta.$$

It is easily seen * that the integral on the right is $\pi\psi(b)$, where

$$\psi(b)=e^{-b}\{(1+2b)I_0+2bI_1(b)\},$$

and I_0 and I_1 are the conjugates of the Bessel functions $\mathcal{J}_0(ib)$ and $\mathcal{J}_1(ib)$. The values of $e^{-b}I_0(b)$ and $e^{-b}I_1(b)$ are given in G. N. Watson's *Bessel Functions*, 698–713, 1922, and thus the values of $\psi(b)$ are easily found.

We now have

$$K_2=\frac{\sqrt{\pi}}{4H^3}e^P\psi(P/2).$$

For a single function of type (7),

$$\bar{\mu}_1=\frac{K_2}{K_1}=\frac{\sqrt{\pi}}{2}\psi(P/2)=\frac{1}{H}G(P),$$

where

$$G(P)=\frac{\sqrt{\pi}}{2}\psi(P/2).$$

The values of $G(P)$ are tabulated in *Groningen Publications*, **30**, 63, 1920, for values of P up to 150, and of $\psi(b)$ in *M.N.*, **95**, 128, 1934, for values of b between 0 and 2.

In the general case,

$$\bar{\mu}_1=\frac{\sum\frac{A\beta^2}{H^3}e^PG(P)}{\sum\frac{A\beta^2}{H^2}e^P}.$$

This method enables us to obtain the correction, for any given value of the observed total proper motion, due to accidental errors.

6. In correcting the statistics of Tables I–III, I have used the first method (section 4) only. We require the probable error r, in either co-ordinate, of the measured components of proper motion. For Group II with the long interval of twenty-five years for the determination of the proper motions, r has been estimated to be $\pm 0''\cdot3$; for the remaining groups the value is $\pm 0''\cdot4$. For reasons to be given later, the adopted values of the probable error r are $0''\cdot4$ for Group II and $0''\cdot5$ for the remaining groups.

Table IV gives the values of the function $\Phi(\mu_0)\equiv e^{-k^2\mu_0^2}$ for a sufficient range in the values of k and of μ_0.

[TABLE IV

* *M.N.*, **95**, 127, equations (55) and (58), 1934.

TABLE IV

The Function $\Phi \equiv e^{-k^2\mu_0^2}$

μ_0 \ k	0·1	0·2	0·3	0·4	0·5	0·6
0·0	1·000	1·000	1·000	1·000	1·000	1·000
0·5	0·997	0·990	0·978	0·961	0·939	0·914
1·0	0·990	0·961	0·914	0·852	0·779	0·698
1·5	0·978	0·914	0·817	0·698	0·570	0·445
2·0	0·961	0·852	0·698	0·527	0·368	0·237
2·5	0·939	0·779	0·570	0·368	0·210	0·105
3·0	0·914	0·698	0·445	0·237	0·105	0·039
3·5	0·885	0·613	0·332	0·141	0·047	0·012
4·0	0·852	0·527	0·237	0·077	0·018	0·003
5·0	0·779	0·368	0·105	0·018	0·002	...
6·0	0·698	0·237	0·039	0·003	...	...
7·0	0·613	0·141	0·012	0·001	...	...
8·0	0·527	0·077	0·003	...	...	...

Table V gives the values of β calculated from (8) for a range in r from 0″·2 to 1″·0.

TABLE V

Values of β

k \ r	0″·2	0″·3	0″·4	0″·5	0″·6	0″·7	0″·8	0″·9	1″·0
0·1	0·100	0·100	0·100	0·100	0·101	0·101	0·102	0·102	0·102
0·2	0·201	0·201	0·203	0·205	0·207	0·209	0·213	0·216	0·220
0·3	0·303	0·306	0·311	0·316	0·324	0·334	0·347	0·364	0·386
0·4	0·405	0·413	0·426	0·441	0·463	0·494	0·539	0·609	0·731
0·5	0·512	0·527	0·552	0·593	0·643	0·737	0·920	1·506	...
0·6	0·620	0·649	0·694	0·771	0·914	...	...	...	...
0·7	0·732	0·784	0·865	1·030	...	...	...	...	...

Generally, the observed distribution shows a much greater proportion of the larger proper motions than is allowed for by the distribution according to formula (7) or (10). By omitting several of the larger proper motions (the number is found by trial and error) the remaining distribution gives, as a rule, a sufficiently satisfactory representation between 0″·0 and 4″·0 of the function Φ or of a combination of such functions. Let N be the total number of stars and N' the number in the adjusted distribution. Let $\bar{\mu}_0$ now denote the mean observed value of the N total proper motions —it is obtained from the last two lines of Tables I – III. Then the corrected mean total proper motion, $\bar{\mu}$, to be used in formula (1) is given by

$$\bar{\mu} = \bar{\mu}_0 - \frac{N'}{N}c, \tag{21}$$

where c is the correction given by (11)—or by (12).

In this way we correct the great majority of the proper motions for accidental error, leaving uncorrected a few of the larger ones and also the residual numbers between the theoretical and observed distributions. The

mean observed proper motion is thus somewhat under-corrected, and to make up for this the adopted values of the probable error r are taken to be a little greater than the values estimated from the plate-measures. As already stated, the values of r used in the subsequent calculations are $0''\cdot4$ for Group II and $0''\cdot5$ for the rest of the groups.

We illustrate the method taking the data of the Group II stars of magnitude 9·0–9·4. The numbers of observed proper motions between $0''\cdot0$ and $0''\cdot5$, $0''\cdot5$ and $1''\cdot0$, etc., up to $4''\cdot0$ are given in the second column of Table VI—these numbers are taken from Table II. The number of proper motions greater than $4''\cdot0$ is assumed to be 5, giving a total $N' = 100$ for the distribution. Thus 15 out of the 20 proper motions greater than

TABLE VI

Derivation of k for Group II ($9^m\cdot0$–$9^m\cdot4$)

μ_0	No. (observed)	$\frac{1}{N'}N(\mu_0)$	Φ ($k=0\cdot48$)
$''$			
0·0		1·00	1·00
	5		
0·5		0·95	0·94
	12		
1·0		0·83	0·80
	26		
1·5		0·57	0·59
	16		
2·0		0·41	0·40
	18		
2·5		0·23	0·24
	$8\frac{1}{2}$		
3·0		0·14	0·12
	$7\frac{1}{2}$		
3·5		0·07	0·06
	2		
4·0		0·05	0·03
> 4·0	5		

$4''\cdot0$ are unaccounted for in this distribution. In the third column we have the proportion of observed proper motions greater than the corresponding value of μ_0 for the adjusted distribution, $N(\mu_0)$ denoting the number of stars with proper motions of μ_0 and greater. In the last column are the values of Φ for $k=0\cdot48$—this is most readily obtained from graphs of the function Φ for various values of k. The last two columns are in close agreement and, accordingly, the distribution of our 100 stars is well represented by the theoretical function Φ.

With $k=0\cdot48$ and $r=0''\cdot4$, we find from Table V that $\beta=0\cdot525$; formula (11) then gives $c=0''\cdot158$. Also the mean observed value $\bar{\mu}_0$ for the 115 stars is $297''\cdot3/115$, *i.e.* $\bar{\mu}_0=2''\cdot585$. Hence by (21)

$$\bar{\mu}=2''\cdot45,$$

and this is the value to be used in formula (1).

For distributions with a small number of stars, the accordance between the adjusted and the theoretical distributions is not so close as in the example just considered.

Table VII gives the adopted values of N', k and β, and also the galactic latitudes of the regions. For Group I (all magnitudes) the adjusted distribution is between 0″·0 and 3″·0; for all the others it is between 0″·0 and 4″·0 as in the example above.

TABLE VII

Values of N', k, β and the Galactic Latitude

Group	$7^m\cdot0$–$8^m\cdot9$			$9^m\cdot0$–$9^m\cdot4$			$9^m\cdot5$			Galactic Latitude
	N'	k	β	N'	k	β	N'	k	β	
										°
I	25	0·50	0·593	50	0·48	0·562	36	0·46	0·532	−11
II	70	0·48	0·525	100	0·48	0·525	65	0·51	0·564	−10
III	38	0·50($\frac{1}{2}$) 0·20($\frac{1}{2}$)	0·593 0·205	60	0·43	0·485	32	0·60	0·771	−12
IV	34	0·54	0·658	77	0·50	0·593	79	0·55($\frac{4}{5}$) 0·20($\frac{1}{5}$)	0·678 0·205	+ 5
V	13	0·50($\frac{2}{5}$) 0·20($\frac{3}{5}$)	0·593 0·205	20	0·30	0·316	19	0·30	0·316	+62
VI	37	0·50	0·593	82	0·60	0·772	57	0·58	0·732	+ 2
VII	39	0·50	0·593	45	0·55	0·676	34	0·58	0·732	0
VIII	17	0·40	0·443	42	0·48	0·562	37	0·48	0·562	−31

For Group VIII ($7^m\cdot0$–$8^m\cdot9$) the value of k is assumed to be 0·40 and the correction found for $N' = N = 17$.

In Group VII ($9^m\cdot0$–$9^m\cdot4$), in deriving N', 5 stars with μ_0 between 0″·0 and 0″·5 were omitted in forming the adjusted distribution.

In several groups the observed distribution could be represented by means of two theoretical distributions; the corresponding values of k_1 and k_2 are given together with the proportion (shown in brackets) of stars in each of these distributions.

The very large proper motions always give a certain amount of trouble, as, with a comparatively small number of stars, their inclusion or rejection affects the mean $\bar{\mu}_0$ considerably. Most of the fields measured are parallax fields in many of which the parallax star was presumably selected for observation because of its large proper motion. Such stars are not included in the statistics of Tables I–III. There are only three stars with proper motions exceeding 20″ per century, one in each of Groups II, III and VI, and all in the magnitude division $7^m\cdot0$–$8^m\cdot9$. These stars have been given half-weight in forming the means $\bar{\mu}_0$.

7. In Table VIII the mean parallaxes for B.D. magnitudes $7^m\cdot0$–$8^m\cdot9$ are given in the last column. These have been calculated by means of formula (2), the corrected values of $\bar{\mu}$ being given in the second column and the values of D, obtained from (3), in the fourth column. The fifth column contains the mean magnitude $\bar{m}$ on the Harvard scale, the corrections to

the mean B.D. magnitude being obtained from Kapteyn and van Rhijn's table already referred to.

Tables IX and X are arranged in a similar way except that the values of D are not repeated.

In Table X the magnitude correction is taken to be $0^m.3$.

TABLE VIII

Mean Parallaxes (*B.D.* $7^m.0$–$8^m.9$)

Group	$\bar{\mu}$	Number of Stars		$\bar{m}$ (H.S.)	p
	″			m	″
I	2·402	32	1·563	8·5	0·00372
II	2·903	83	1·582	8·5	444
III	3·059	38	1·516	8·5	489
IV	2·179	38	1·344	8·6	392
V	4·588	13	1·582	8·3	702
VI	2·348	40	1·323	8·5	430
VII	2·512	45	1·517	8·5	402
VIII	3·150	17	1·615	8·4	472

TABLE IX

Mean Parallaxes (*B.D.* $9^m.0$–$9^m.4$)

Group	$\bar{\mu}$	Number of Stars	$\bar{m}$ (H.S.)	p
	″		m	″
I	2·480	59	9·5	0·00384
II	2·448	115	9·5	375
III	2·568	68	9·4	411
IV	1·933	82	9·5	348
V	4·035	26	9·4	617
VI	1·591	86	9·5	291
VII	1·591	56	9·5	254
VIII	2·213	46	9·5	332

TABLE X

Mean Parallaxes (*B.D.* $9^m.5$)

Group	$\bar{\mu}$	Number of Stars	$\bar{m}$ (H.S.)	p
	″			″
I	1·959	41	9·8	0·00304
II	2·072	68	,,	317
III	1·673	36	,,	268
IV	1·857	84	,,	334
V	3·712	23	,,	568
VI	1·711	60	,,	313
VII	1·803	38	,,	289
VIII	1·891	39	,,	284

8. In Table XI the results in the previous three tables are grouped together according to galactic latitude. The grouping is as follows: $+2°$,

Groups IV, VI and VII ; $-11°$, Groups I, II, III ; $-31°$, Group VIII ; $+62°$, Group V.

The entries within parentheses are the corresponding values of the mean parallaxes taken from Table XXVI of *Groningen Publications*, 29.

TABLE XI

Mean Parallaxes according to Magnitude and Galactic Latitude

Galactic Latitude	$\bar{m}$ (H.S.)	No. of Stars	p	$\bar{m}$ (H.S.)	No. of Stars	p	$\bar{m}$ (H.S.)	No. of Stars	p
°	m		″	m		″	m		″
+ 2	8·5	123	0·00408	9·5	224	0·00303	9·8	182	0·00316
			(503)			(364)			(330)
−11	8·5	153	440	9·5	242	387	9·8	145	301
			(513)			(374)			(338)
−31	8·4	17	472	9·5	46	332	9·8	39	284
			(578)			(411)			(374)
+62	8·3	13	702	9·4	26	617	9·8	23	568
			(753)			(530)			(469)

On the whole, my results are somewhat smaller than those of Kapteyn and van Rhijn, and assuming that the Cambridge stars are fairly representative of the whole sky, the discrepancies could be possibly explained as due partly to over-correction of the Cambridge proper motions and partly to under-correction of the proper motions used in preparing the Groningen table of parallaxes. For the stars of Group II (the most numerous group) a reduction of the probable error to $\pm 0''{\cdot}3$, for example, would result in an increase of the mean parallax by about 3 per cent., whereas an increase in the probable error to $\pm 0''{\cdot}5$ would result in a decrease of the parallax by about 4–5 per cent. It is clear that for probable errors r of order $\pm 1''{\cdot}0$, a small change in r will result in an appreciable alteration of the correction to be applied to the mean observed proper motion and consequently to the calculated mean parallax. From the point of view of correcting statistics as in this paper, the determination of the probable error of the proper motions assumes an importance that cannot be too greatly emphasised.

Observatory, Cambridge :
1935 *December* 7.

MULTIPLET INTENSITIES IN THE SUN AND IRON ARC.

C. W. Allen, M.Sc. (Hackett Research Student).

(Communicated by Professor F. J. M. Stratton)

Introduction.—The intensities of lines in the Sun's spectrum while in agreement with theory for some multiplets are in disagreement for others, and in a recent publication * the view was expressed that some of the disagreement was due to failure of the theoretical intensity formulæ. This conclusion was based chiefly on the fact that when laboratory intensity measures of arc spectra disagreed with theory (worked out on a basis of *LS* coupling) there was a tendency for solar intensities to follow the arc values rather than theory. The evidence, however, was not very conclusive, and hence five more multiplets in the iron arc have been measured for comparison with the Sun. The solar lines of all five multiplets show wide departure from theory, and a similar departure has now been found in the arc intensities.

Measurements.—Using a three-ampere iron arc, spectra were obtained in the first order of the 21-foot Rowland grating at the Solar Physics Observatory, Cambridge. Ilford Special Rapid Panchromatic plates were employed, and a portion of each plate standardised by means of an optical wedge attached to a small auxiliary spectrograph. The colour sensitivity correction was determined by photographing a number of solar spectra on one of the plates alongside the iron spectrum, and using the known distribution of light in the background of the solar spectrum.† The plates were photometered with a Cambridge microphotometer, and the intensities reduced in the usual way. Large background corrections were necessary for many of the faint lines. There was a certain amount of blending in the arc, and the sharpness of the lines made for difficult photometry. Hence high accuracy is not to be expected, but it is not required for the present purpose. In order to apply an excitation correction an excitation temperature of 6000° K. was assumed for the arc, but the corrections within each multiplet were small and any error in this assumed temperature is quite insignificant. Finally the values were multiplied by λ^4 in order to obtain experimental line strengths from the actual intensities.

Results.—The results are given in Table I, where they are compared with the line strengths determined from solar intensity data (taken from my earlier work ‡ in which the symbol $N_z \cdot f$ is analogous to line strength), and also with the theoretical line strengths for *LS* coupling (called sum rule intensity in the earlier paper). The absolute values are quite arbitrary,

* C. W. Allen, *Mem. C.S.O. Canberra*, No. 5, 1934.

† G. F. W. Mulders, "Aequivalente breedten van Fraunhoferlijnen in het zonnespectrum," Dissertation, Utrecht, 1934.

‡ *Loc. cit.*

TABLE I

Table of logarithms of line strengths derived from the arc, the Sun, and theory (LS coupling); and the deviations showing the comparison of these sources of line strength.

Multiplet	λ	Log Strength (Arc)	Log Strength (Sun)	Log Strength (Theory)	Deviations from the Mean of Log Arc/Sun	Log Arc/Theory	Log Sun/Theory
(1) $z^3D^\circ - e^3D$	4896	0·67	4·59	1·59	−0·06	−0·71	−0·65
	4911	1·00	4·92	1·57	−0·06	−0·34	−0·30
	4973	1·95	5·92	2·05	−0·11	0·11	0·22
	4985	2·35	6·16	2·24	0·05	0·32	0·27
	5005	2·70	6·45	2·49	0·11	0·42	0·31
	5048	1·60	5·37	1·57	0·09	0·24	0·15
		Average deviations			0·08	0·36	0·32
(2) $z^3F^\circ - e^3D$	5001	3·80	6·60	2·65	0·00	0·25	0·25
	5014	3·64	6·38	2·49	0·06	0·25	0·19
	5022	3·37	6·11	2·32	0·06	0·15	0·09
	5099	2·45	5·19	1·59	0·06	−0·04	−0·10
	5129	1·88	4·88	1·59	−0·20	−0·61	−0·41
		Average deviations			0·08	0·26	0·20
(3) $a^3F - z^3F^\circ$	5041	3·05	4·63	0·60	−0·15	0·57	0·72
	5107	2·92	4·34	0·59	0·01	0·45	0·44
	5171	3·37	4·66	1·77	0·14	−0·28	−0·42
	5194	3·25	4·42	1·61	0·26	−0·24	−0·50
	5216	3·15	4·33	1·49	0·25	−0·22	−0·47
	5307	2·27	4·00	0·59	−0·30	−0·20	0·10
	5332	2·42	4·06	0·60	−0·21	−0·06	0·15
		Average deviations			0·19	0·29	0·40
(4) $y^5D^\circ - f^5D$	5353	5·11	5·90	0·88	0·00	0·31	0·32
	5391	5·02	5·65	1·00	0·16	0·10	−0·05
	5473	5·27	5·90	1·24	0·16	0·11	−0·04
	5476	5·66	6·66	1·57	−0·21	0·17	0·39
	5478	4·05	4·96	0·80	−0·12	−0·67	−0·54
	5480	4·77	5·53	0·70	0·01	0·15	0·13
	5493	3·45	4·41	0·10	−0·17	−0·57	−0·39
	5525	4·74	5·49	0·70	0·04	0·12	0·09
	5543	4·83	5·61	0·94	0·01	−0·03	−0·03
	5563	5·21	5·85	1·00	0·15	0·29	0·15
		Average deviations			0·10	0·25	0·21

Multiplet	λ	Log Strength (Arc)	Log Strength (Sun)	Log Strength (Theory)	Deviations from the Mean of		
					$\text{Log}\frac{\text{Arc}}{\text{Sun}}$	$\text{Log}\frac{\text{Arc}}{\text{Theory}}$	$\text{Log}\frac{\text{Sun}}{\text{Theory}}$
(5) $y^3F^\circ - f^3D$	5554	6·54	6·28	1·65	−0·02	−0·43	−0·41
	5565	6·64	6·58	1·49	−0·22	−0·17	0·05
	5598	6·60	6·18	1·32	0·14	−0·04	−0·18
	5679	6·07	5·84	0·59	−0·05	0·16	0·21
	5706	6·41	5·98	0·59	0·15	0·50	0·35
		Average deviations			0·12	0·26	0·24

and except for the Sun are not comparable in different multiplets. The last three columns give the deviations of the log strength ratios from the mean value for the multiplet, and hence show the quality of the agreement of (*a*) the arc with the Sun, (*b*) the arc with theory, and (*c*) the Sun with theory. From the average deviations at the foot of these columns it is seen at once that the agreement between the Sun and arc is considerably better than the

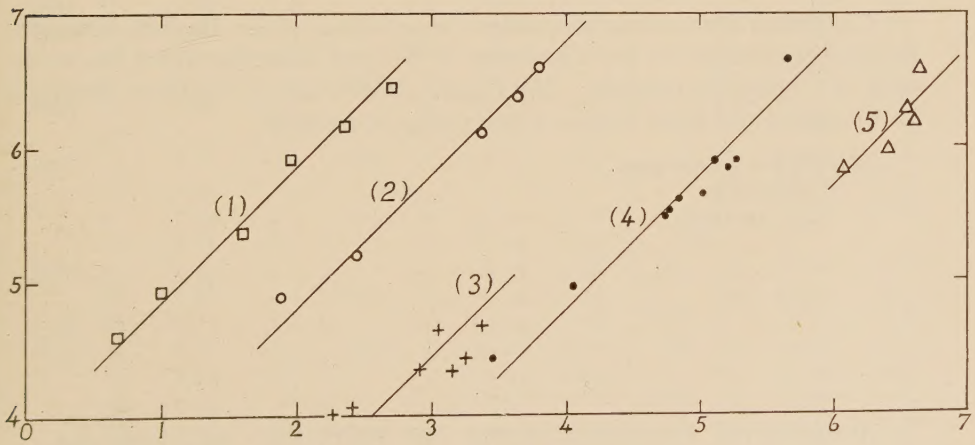

FIG. 1.—*Log Arc Line Strength.*

agreement of theory with either Sun or arc. The multiplets (1), (2), (4) and (5) show an arc-Sun agreement well within experimental error, but for multiplet (3) there seems to be some other factor at work. Even here, however, the disagreement may be largely experimental, and it is not as great as the Sun-theory or arc-theory disagreement.

The results have been plotted in figs. 1 and 2, which show that the arc intensity measures correspond with the solar intensities much better than with theory. Hence it may be presumed that the arc and the Sun both represent true line strengths. The great values of the solar intensities lies

in the fact that they represent a very wide range of line strength, over a wide wave-length region, and in a practically constant light source. Where

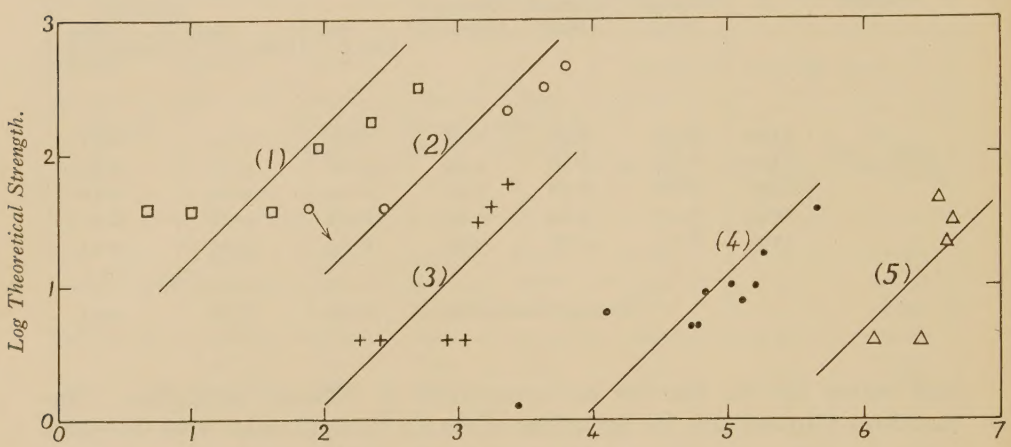

FIG. 2.—*Log Arc Line Strength.*

individual line accuracy is not required they should provide suitable data for testing intensity calculations in complex spectra.

The author wishes to acknowledge the assistance of the Hackett Research Studentship granted by the University of Western Australia which has made work at Cambridge possible. His thanks are also due to Professor Stratton for the use of the Solar Physics Observatory equipment.

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HORNSBY'S OBSERVATIONS OF SATURN, 1774–1779 and 1790–1797.

Bernard Francis Bawtree.

(Communicated by H. Knox-Shaw)

The following are comparisons of Hornsby's observations of Saturn, with those obtained from Newcomb's Tables. The observations were published in *The Observations of the Reverend Thomas Hornsby* (Oxford, 1932). Newcomb's Tables were published in vol. vii of *Astronomical Papers of the American Ephemeris.* Similar results for the period 1780–1784 will be found in the volume already referred to.

Saturn

Observed *minus* Computed

Date	R.A.	Dec.
1774	s	″
Jan. 23	−0·01	−3·2
Mar. 13	−0·09	+4·3
28	−0·10	+3·2
May 1	−0·29	+0·5
10	−0·11	−2·5
13	−0·04	+1·1
23	−0·13	−1·8
June 1	−0·11	+0·4
7	−0·22	−0·3
13	−0·12	−1·7
18	+0·05	+1·1
July 14	−0·08	−1·9
Aug. 7	−0·19	−3·6
Nov. 30	−0·11	−0·2
Dec. 6	−0·09	−4·7
12	−0·07	+0·3
24	−0·30	−5·6
30	−0·10	−2·2
1775		
Mar. 26	−0·06	−1·5
Apr. 13	+0·09	+0·4
19	−0·05	+0·7
20	−0·19	+2·1
May 10	+0·06	+2·0
30	−0·10	−1·2
June 4	−0·16	−2·6
18	−0·29	−2·5
21	−0·22	−4·9
25	+0·02	−0·1
July 20	−0·28	−2·0
26	−0·26	−1·4
Aug. 10	−0·29	−8·4
Dec. 5	−0·17	+2·5
8	−0·33	+2·6
9	−0·03	+2·8
18	−0·18	+3·6
26	−0·02	−0·3
1776		
Mar. 30	−0·17	−0·5
May 28	−0·03	−3·8
June 7	+0·03	+0·1
15	−0·33	−4·3
16	−0·19	+0·2
18	−0·23	−2·8
19	−0·24	−3·4
22	−0·27	−1·4
July 13	−0·12	−1·4
14	−0·34	−1·8
22	−0·23	+0·1
1776		
July 26	−0·11	−4·2
Dec. 25	−0·11	−2·1
1777		
Apr. 19	−0·10	+4·1
May 16	−0·07	+5·2
26	−0·15	+0·2
July 13	−0·27	+5·6
14	−0·11	−4·9
16	−0·29	−1·9
1778		
Jan. 25	+0·13	+0·1
May 4	−0·11	−0·2
21	−0·01	+0·5
27	−0·07	+1·5
June 1	−0·06	0·0
2	+0·14	+0·2
8	+0·04	+1·0
10	−0·02	+3·2
15	−0·09	+1·9
16	+0·08	+2·7
23	−0·09	+2·2
28	0·00	+0·2
July 6	−0·11	+2·2
8	−0·16	−0·3
14	−0·02	−1·2
Aug. 7	−0·13	−7·2
1779		
Jan. 1	−0·29	−2·1
11	−0·04	−2·4
13	+0·06	+2·6
Feb. 1	−0·18	−2·8
17	+0·04	+0·3
May 5	+0·05	+2·5
11	+0·01	+2·1
17	+0·11	+1·8
June 23	+0·05	−1·4
1790		
Jan. 7	+0·07	−2·4
14	−0·13	−0·8
15	−0·07	−5·6
18	−0·08	−4·0
Sept. 25	−0·07	−1·4
Oct. 10	−0·18	+0·4
14	−0·03	+0·2
24	−0·07	+0·4
25	−0·25	−0·7
31	−0·08	−0·3
Nov. 2	−0·11	−1·8
13	+0·12	−3·9
1790		
Nov. 15	+0·16	−2·2
Dec. 6	−0·03	+1·2
12	+0·09	−0·4
18	+0·06	+1·0
21	+0·14	−3·8
22	−0·17	+0·7
23	−0·05	−1·9
25	−0·01	−0·5
28	−0·19	−3·8
31	+0·01	+0·3
1791		
Jan. 6	+0·02	0·0
8	+0·10	−1·2
9	+0·05	−0·1
17	+0·06	−2·6
19	−0·03	+4·8
20	−0·17	−2·8
Oct. 11	−0·10	−0·8
Nov. 25	−0·04	−1·4
Dec. 10	+0·13	−2·0
13	+0·17	−2·9
17	−0·05	+2·2
23	+0·07	+0·3
24	+0·12	+3·0
25	+0·02	−1·9
26	+0·06	−0·5
28	+0·06	+0·3
29	+0·01	+1·6
1792		
Jan. 3	+0·10	−1·9
12	+0·05	0·0
Oct. 22	−0·12	−1·0
Nov. 12	−0·06	+0·4
16	+0·07	−0·8
Dec. 11	−0·02	−1·4
15	+0·16	+0·8
19	0·00	−2·7
21	+0·18	−3·3
28	+0·18	+2·3
30	+0·15	+2·6
1793		
Jan. 2	+0·15	−3·3
4	−0·11	+1·3
6	−0·01	+1·0
8	+0·03	−4·6
11	+0·14	−1·2
14	−0·02	+2·0
18	−0·01	−1·1
27	+0·02	−4·3

Date	R.A.	Dec.
1793	s	″
Jan. 28	+0·02	+1·6
Feb. 3	+0·06	−2·6
17	−0·19	−1·1
Nov. 19	−0·05	0·0
22	+0·18	−0·4
Dec. 4	−0·08	−0·7
13	+0·10	+2·8
16	−0·03	+1·0
19	−0·05	+5·3
1794		
Jan. 7	−0·07	−0·4
9	−0·01	+0·1
10	−0·09	+2·0
19	−0·05	0·0
23	−0·09	−2·4
25	−0·04	+0·2
27	−0·03	−3·6
28	+0·03	−1·0
Feb. 2	−0·14	+3·0
4	+0·15	−0·1
Aug. 28	+0·18	+1·4
Nov. 27	+0·02	−1·2

Date	R.A.	Dec.
1794	s	″
Dec. 10	−0·15	−0·5
18	−0·04	+1·4
19	−0·08	+2·1
1795		
Jan. 1	−0·05	+1·6
5	−0·02	−0·2
10	−0·20	+0·7
11	−0·23	−3·7
12	−0·01	−2·1
23	+0·06	−0·7
24	−0·03	+0·2
28	+0·03	−2·2
Feb. 5	−0·01	+0·7
11	+0·04	−0·7
13	−0·14	+0·3
14	+0·08	+1·3
Mar. 1	−0·12	+8·6
Aug. 15	−0·14	−1·3
17	+0·36	−1·7
21	+0·03	−5·9
26	−0·01	−2·3
Sept. 2	−0·15	−1·4
11	−0·10	−1·5

Date	R.A.	Dec.
1795	s	″
Sept. 15	−0·02	+1·4
Dec. 14	+0·11	+1·4
22	+0·11	+0·5
30	−0·02	+0·9
1796		
Jan. 29	+0·05	−0·3
Feb. 1	+0·11	+1·1
13	+0·08	+1·0
14	+0·05	−0·8
15	−0·01	+1·7
23	−0·06	+0·7
Mar. 1	+0·03	+2·8
6	+0·07	−1·7
15	−0·17	−4·9
16	0·00	−8·8
19	+0·06	−6·0
Dec. 24	−0·43	−4·8
25	−0·06	−1·8
1797		
Feb. 15	+0·23	+5·4
18	+0·29	−1·5
28	+0·08	−1·0